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The development of convective motion in a bottom heated square enclosure containing ice and water

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Abstract — A numerical study of the steady state flow in a square enclosure with two vertical walls which are adiabatic and with two horizontal isothermal walls has been undertaken. The enclosure contains water and the upper wall is maintained at a uniform temperature that is below the freezing point of water while the lower wall is maintained at a uniform temperature that is above the freezing point of water. The upper portion of the enclosure is thus filled with ice and the lower portion is filled with water. The conditions considered in the present study are such that there can be significant natural convection in the water and the effect of the density maximum that exists in the water at approximately 4 °C can have a significant effect on this flow. The main aim of the study was to determine how far above 4 °C the hot wall temperature can be before significant convective motion develops in the water. The governing equations have been expressed in dimensionless form and solved using a finite element procedure. The effect of the various governing parameters on the mean Nusselt number has mainly been considered and the effect of the lower surface temperature has, in particular, been studied. These results have then been used to determine the conditions under which convective motion develops. © 2001 Éditions scientifiques et médicales Elsevier SAS

melting / natural convection / finite element method

Nomenclature

a'	coefficient in density–temperature relationship	K^{-2}
g'	gravitational acceleration	$m \cdot s^{-2}$
k'	thermal conductivity of water	$W \cdot m^{-1} \cdot K^{-1}$
k_r	ratio of solid to liquid thermal conductivities	
n	$= n'/W'$	
n'	coordinate measured normal to a surface	m
Pr	Prandtl number	
Ra^*	modified Rayleigh number	
T'	temperature	K
T'_C	temperature of top wall	K
T'_F	solidification temperature	K
T'_H	temperature of bottom wall	K
T'_{max}	temperature of maximum density	K
T	dimensionless temperature	
T_C	dimensionless temperature of top wall	
T_H	dimensionless temperature of bottom wall	
T_{HA}	dimensionless temperature of bottom wall at which liquid motion starts	
T_{max}	dimensionless temperature of maximum density	
t'	time	s
t	dimensionless time	
u'	velocity component in x' direction	$m \cdot s^{-1}$
u	dimensionless velocity component in x direction	
v'	velocity component in y' direction	$m \cdot s^{-1}$
v	dimensionless velocity component in y direction	
x'	horizontal coordinate	m
x	dimensionless horizontal coordinate	
y'	vertical coordinate	m
y	dimensionless vertical coordinate	
W'	enclosure size	m
w'_l	thickness of liquid layer in pure conduction case	m
w'_s	thickness of solid layer in pure conduction case	m

Greek symbols

α'	thermal diffusivity	$m^2 \cdot s^{-1}$
ψ'	stream function	$m^2 \cdot s^{-1}$

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ψ	dimensionless stream function	
ω'	vorticity	s^{-1}
ω	dimensionless vorticity	
ρ'	density	$kg \cdot m^{-3}$
ρ'_{max}	maximum density	$kg \cdot m^{-3}$

1. INTRODUCTION

The flow in a square enclosure with two vertical walls which are adiabatic and with two horizontal isothermal walls has been considered. The enclosure contains water and the upper wall is maintained at a temperature that is below the freezing point of water while the lower wall is maintained at a temperature that is above the freezing point of water. The upper portion of the enclosure is thus filled with ice and the lower portion is filled with water. The flow situation considered is shown schematically in *figure 1*.

The present study was undertaken in support of experimental studies of the nature of the ice that forms under various conditions. These studies require that there be no convective motion in the water during the freezing. Because water has a density maximum at approximately 4 °C, the experiments are usually carried out with the hot wall at the bottom of the enclosure and with this wall kept at a temperature below 4 °C. However, this limits the range of conditions that can be covered in the experimental work. The question therefore arises as to how far above 4 °C can the hot wall temperature be before significant convective motion develops in the water. The present numerical study was undertaken, basically, to provide a partial answer to this question.

There have been many previous studies of solidification and melting of liquids in enclosures. Most of these studies have, however, been concerned with the evolution of the flow with time and have not been concerned with a detailed study of the effects of the various governing parameters on the final steady state for the case where there is under-cooling. A review of much of this work is given by Yao and Prusa [1] and Fukusako and Yamada [2]. A numerical study of the particular case of the freezing of pure water in a rectangular enclosure is given by De Vahl Davis et al. [3], this paper also providing a review of past work on the subject. Numerical studies of steady state freezing of water in a rectangular enclosure are described by Oosthuizen [4] and Oosthuizen and Paul [5], for example. Experimental studies of freezing in an enclosure for situations in which the density maximum is important are described by Oosthuizen and Xu [6], Braga and Viskanta [7] and Schutz and Beer [8]. Merker et al. [9] examined the effect of the density maximum on the onset of convection in a horizontal water layer. Their results are not directly comparable to those given here because the presence of an ice layer is considered in the present study.

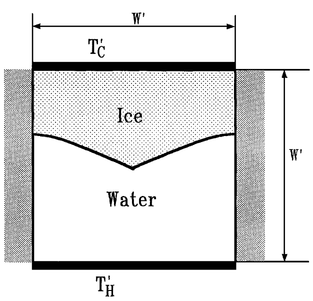

Figure 1. Flow situation considered.

These existing studies do not give much information on the condition under which natural convective motion will be important under the circumstances here being considered.

2. SOLUTION PROCEDURE

It has been assumed that if convective motion develops, the flow remains laminar and can be treated as two-dimensional and that the fluid properties can be assumed constant except for the density change with temperature which gives rise to the buoyancy forces. The relation between the density and the temperature was assumed to be

$$\frac{\rho'_{max} - \rho'}{\rho'} = a'\left(T' - T'_{max}\right)^2 \tag{1}$$

the subscript max referring to conditions at the maximum density temperature.

The solution for the water, in which the natural convection has been assumed to be important, has been obtained in terms of the stream function and vorticity, defined, as usual, by

$$u' = \frac{\partial \psi'}{\partial y'}, \qquad v' = \frac{\partial \psi'}{\partial x'} \tag{2}$$

$$\omega' = \frac{\partial v'}{\partial x'} - \frac{\partial u'}{\partial y'} \tag{3}$$

The prime (′) denotes a dimensional quantity.

The unsteady form of the governing equations have been written in dimensionless form, the following dimensionless variables being introduced for this purpose:

$$\psi = \frac{\psi'}{\alpha'}, \qquad \omega = \frac{\omega' W'^2}{\alpha'} \tag{4}$$

$$x = \frac{x'}{W'}, \qquad y = \frac{y'}{W'} \tag{5}$$

$$t = \frac{t'}{\alpha' W'^2} \tag{6}$$

$$T = \frac{T' - T'_F}{T'_H - T'_C} \tag{7}$$

where α' is the thermal diffusivity of the water and T'_F is the freezing temperature.

In terms of these dimensionless variables, the governing equations for the liquid flow are:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega \tag{8}$$

$$\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right) - \frac{1}{Pr}\left(\frac{\partial \psi}{\partial y}\frac{\partial \omega}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \omega}{\partial y} + \frac{\partial \omega}{\partial t}\right) = -2Ra^* T \frac{\partial T}{\partial x} \tag{9}$$

$$\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) - \left(\frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y} + \frac{\partial T}{\partial t}\right) = 0 \tag{10}$$

Here Ra^* is the modified Rayleigh number defined by

$$Ra^* = \frac{a' g' W'^3 (T'_H - T'_C)^2}{\nu' \alpha'} \tag{11}$$

The equation governing the temperature distribution in the solid phase is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{\partial T}{\partial t} \tag{12}$$

The boundary conditions on the solution are as follows.

- On all solid surfaces:

$$\psi = 0, \qquad \frac{\partial \psi}{\partial n} = 0 \tag{13}$$

where n is the coordinate measured normal to the surface considered.

- On vertical side walls:

$$\frac{\partial T}{\partial x} = 0 \tag{14}$$

- On the upper surface:

$$T = T_C \tag{15}$$

- On the bottom surface:

$$T = T_H (= 1 + T_C) \tag{16}$$

- On the interface between the water and ice the following conditions apply:

$$\psi = 0, \qquad \frac{\partial \psi}{\partial n} = 0, \qquad T = 0$$
$$\left.\frac{\partial T}{\partial n}\right|_l = \left(\frac{k_s}{k_l}\right)\left.\frac{\partial T}{\partial n}\right|_s \tag{17}$$

where the subscripts l and s refer to conditions on the liquid and solid sides of the interface, respectively.

The above dimensionless equations, i.e. equations (8)–(10) and (12), have been solved using a finite element procedure. The solution was started by assuming that there is no motion in the water layer and that the one-dimensional pure conduction solution applies. The solution was continued in dimensionless time for a sufficiently long dimensionless time to ensure that either no convective motion would develop or until a steady state flow had developed. As the solution progresses, the interface position is locally modified according to the difference between the calculated rates of heat transfer at the interface on the solid and liquid sides, the element shapes being adaptively modified to follow the changing interface shape.

The solution for the temperature distribution allows the local heat transfer rate over the upper and lower surfaces to be determined. The local heat transfer rate distribution can then be integrated to give the mean heat transfer rates for the upper and lower surfaces. The mean heat transfer rate has been expressed in terms of the mean Nusselt number, Nu, based on the overall temperature difference and on the enclosure size, i.e.

$$Nu = \frac{q' W'}{k'(T'_H - T'_C)} \tag{18}$$

Calculations have been carried out with various nodal distributions and the results obtained indicate that the results presented in this paper are grid independent to better than 1 %.

Now under some conditions there is no convective motion in the liquid, the basic purpose of this study being to determine when this is the case. In this situation the interface between the solid and the liquid is flat and parallel to the top and bottom walls (*figure 2*) and the heat transfer in both the solid and the liquid is by conduction so that:

$$q' = \frac{k'(T'_H - T'_F)}{w'_l} = \frac{k_r k'(T'_F - T'_C)}{w'_s} \tag{19}$$

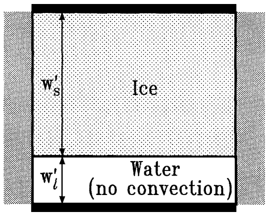

Figure 2. Situation when there is no convective motion in the liquid.

where w'_l and w'_s are the thickness of the liquid and solid layers as indicated in *figure 2*.

Hence, since

$$w'_l + w'_s = W'$$

it follows that

$$\frac{k'(T'_H - T'_F)}{q'} + \frac{k_r k'(T'_F - T'_C)}{q'} = W'$$

which can be rearranged to give

$$\frac{q'W'}{k'(T'_H - T'_C)} = \frac{T'_H - T'_F}{T'_F - T'_C} + k_r \frac{T'_F - T'_C}{T'_H - T'_C}$$

i.e. assuming $k_r = 4$

$$Nu = T_H - 4T_C \tag{20}$$

But it was noted before that

$$T_H - T_C = 1$$

Hence, when there is no motion in the liquid

$$Nu = 4 - 3T_H \tag{21}$$

In addition to the calculations described above, that account for the maximum density effect, some calculations were also undertaken for the case where there is no density maximum. In this case the relation between the density and the temperature was assumed, as usual, to be

$$\frac{\rho'_C - \rho'}{\rho'} = \beta'\left(T' - T'_C\right) \tag{22}$$

the subscript C referring to conditions at the cold temperature. In this situation the conventional Rayleigh number, i.e.

$$Ra = \frac{\beta' g' W'^3 (T'_H - T'_C)}{\nu'^2} \tag{23}$$

is the parameter on which the flow depends.

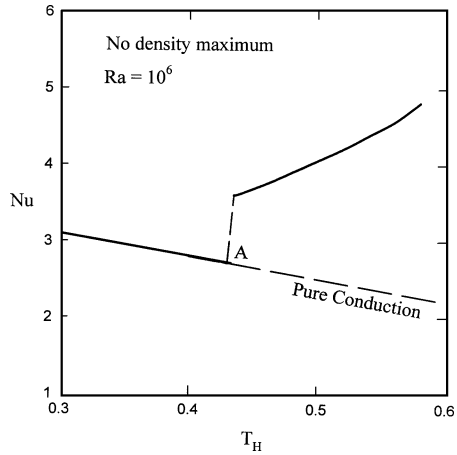

Figure 3. Typical variation of mean Nusselt numbers with dimensionless hot wall temperature for the case where there is no density maximum.

3. RESULTS

The solution has the following parameters:

- the modified Rayleigh number, Ra^*;
- the Prandtl number, Pr;
- the ratio of the thermal conductivity of the solid to that of the liquid, k_r;
- the dimensionless temperature of the hot wall, T_H;
- the dimensionless temperature at which the maximum density occurs, T_{max}.

Results have been obtained here for a Prandtl number of 12 and a conductivity ratio, k_r, of 4. The value of the dimensionless maximum density temperature, T_{max}, basically will be determined by the difference between the hot and cold wall temperatures because $T'_{max} - T'_F$ is approximately 4 °C for water, T'_F being the solidification temperature.

Results for the case where there is no density maximum will first be considered. *Figure 3* shows a typical variation of Nusselt number with dimensionless hot wall temperature for this case. Also shown in this figure is the variation that applies when there is no convective motion in the liquid, i.e. when the heat transfer is by pure conduction. It will be seen that at low values of the dimensionless hot wall temperature there is no motion in the liquid. However, at the point indicated as A in *figure 3*, significant motion develops with the result that the thickness of the ice layer decreases and that of the water layer increases which leads to an intensification of the liquid motion. As a result, there is a sharp increase in Nusselt number at point A. The dimensionless hot wall temperature at which significant convective motion commences will here be termed T_{HA}.

Now it follows from equation (19) that when there is no convective motion in the liquid:

$$w'_l = \frac{k'(T'_H - T'_F)}{q'}$$

Hence, when there is no liquid motion:

$$\frac{w'_l}{W'} = \frac{k'(T'_H - T'_C)}{q'W'} \frac{T'_H - T'_F}{T'_H - T'_C} = \frac{T_H}{Nu}$$

and the Rayleigh number Ra_l based on the difference between the hot wall and freezing temperatures is given by

$$Ra_l = Ra\left(\frac{w'_l}{W'}\right)^3 \frac{T'_H - T'_F}{T'_H - T'_C} = Ra\left(\frac{T_H}{Nu}\right)^3 T_H$$

i.e.

$$Ra_l = Ra\,\frac{T_H^4}{(4-3T_H)^3}$$

Now it will be seen from *figure 3* that convective motion commences when T_{HA} is approximately equal to 0.43. Because the overall Rayleigh number is 10^6, this indicates that convective motion in the liquid commences when Ra_l is approximately equal to 1 720. This is effectively the same as the value at which Rayleigh–Bénard convection commences between horizontal heated and cooled surfaces which is to be expected.

Attention will now be turned to the case where the density maximum is accounted for. *Figure 4* shows a typical variation of Nusselt number with dimensionless hot wall temperature. It will be seen that even at dimensionless temperatures well above the dimensionless maximum density temperature there is no significant convective motion. However, at the point indicated as A in *figure 4*, significant motion develops with the result that the thickness of the ice layer decreases and that of the water layer increases which leads to an intensification of the liquid motion. As a result, there is a sharp increase in Nusselt number at point A. This behavior is, of course, basically the same as that observed when there is no density maximum. The dimensionless hot wall temperature at which significant convective motion commences will, as discussed before, be designated as T_{HA}.

The basic aim of the present study is, of course, to determine T_{HA} since below this value of the dimensionless wall temperature there is no motion in the liquid.

The sharp changes that occur when the convective motion commences in the liquid can further be illustrated by considering the variation of the mean dimensionless liquid layer thickness, $w_l = w'_l/W'$, with T_H. A typical variation is shown in *figure 5*. Also shown in this figure is a typical variation for the case where maximum density effects are negligible and the variation that would occur if there was no liquid motion, this variation being given by the analysis presented above by

$$w'_l = \frac{T_H}{4-3T_H}$$

It will be seen from the results given in *figure 5* that there is a very rapid increase in the liquid layer thickness when convective motion commences in the liquid layer.

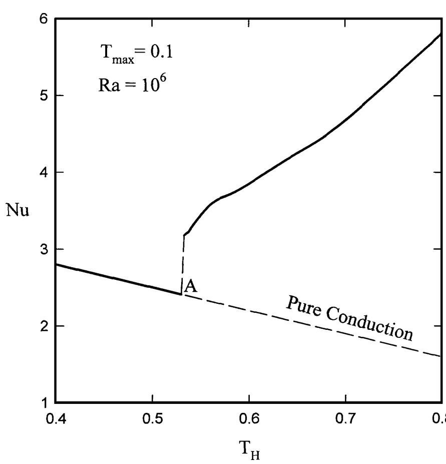

Figure 4. Typical variation of mean Nusselt numbers with dimensionless hot wall temperature.

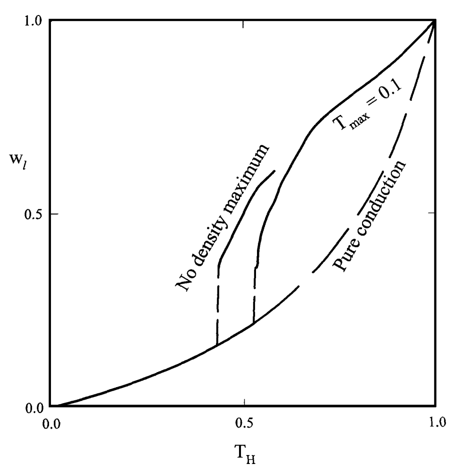

Figure 5. Typical variations of mean liquid layer thickness with dimensionless hot wall temperature.

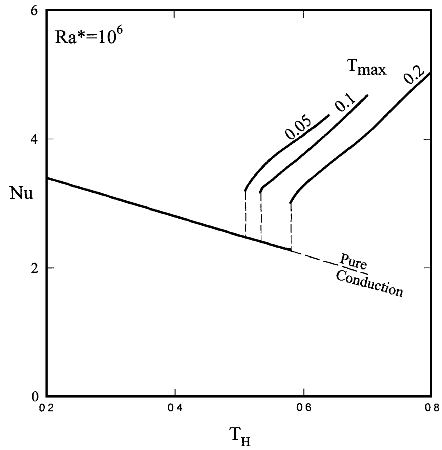

Figure 6. Typical variations of mean Nusselt numbers with dimensionless hot wall temperature for various values of the dimensionless maximum density temperature.

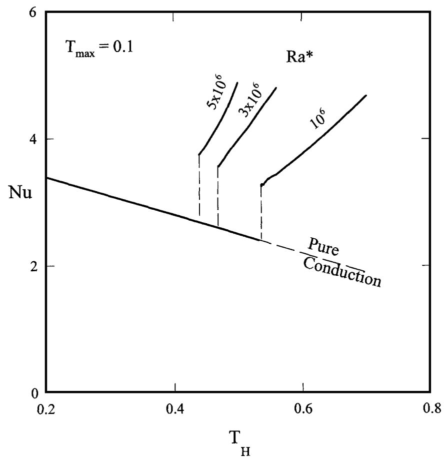

Figure 7. Typical variations of mean Nusselt numbers with dimensionless hot wall temperature for various values of the modified Rayleigh number.

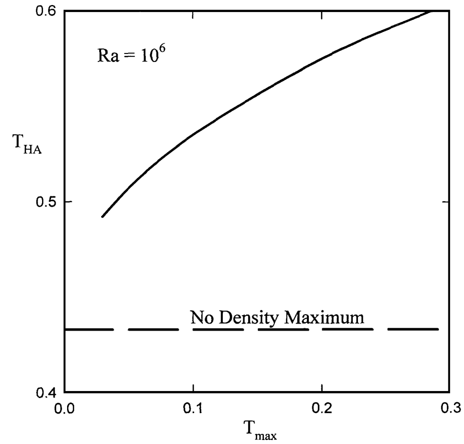

Figure 8. Typical variation of dimensionless hot wall temperature at which convective motion commences with dimensionless maximum density temperature.

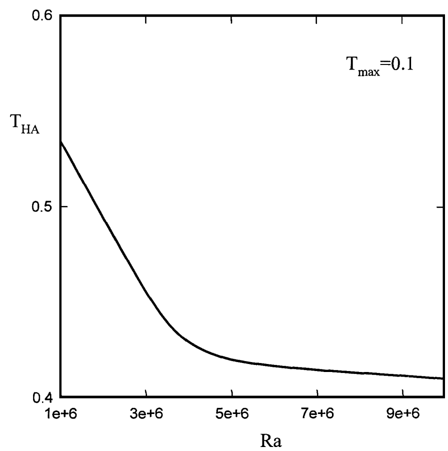

Figure 9. Typical variation of dimensionless hot wall temperature at which convective motion commences with modified Rayleigh number.

The effect of dimensionless maximum density temperature and of modified Rayleigh number on the value of T_{HA} has been studied. Typical variations of Nusselt number with dimensionless hot wall temperature for various values of T_{max} are shown in *figure 6* and for various values of Ra^* are shown in *figure 7*.

Using results such as these, the variations of T_{HA} with T_{max} and Ra^* have been determined. A typical variation of T_{HA} with dimensionless maximum density temperature is shown in *figure 8*, the value of T_{HA} for the case where there is no density maximum also being shown for comparison. Similarly, a typical variation of T_{HA} with Rayleigh number is shown in *figure 9*.

The results given in *figures 8* and *9* clearly show that convective motion does not develop in the liquid until the dimensionless hot wall temperature is considerably higher than the dimensionless maximum density temperature. These figures together can be used to estimate when the liquid motion will occur.

4. CONCLUSIONS

The results of the present study indicate that convective motion does not occur until the lower hot wall temperature is well above the maximum density temperature. The results given by the study can be used to determine the actual temperature at which significant convective motion will develop.

Acknowledgement

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